AD-A172 749 ON THE PENETRATION OF SEMI-INFINITE TARGETS BY LONG RODS(U) KENTUCKY UNIV LEXINGTON S E JONES ET AL. AUG 86 AFATL-TR-86-56 F08635-82-K-0324 1/1

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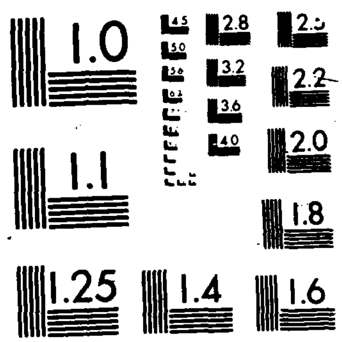
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AFATL-TR-86-56

On the Penetration of Semi-Infinite Targets by Long Rods

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LEXINGTON, KENTUCKY 40506

AUGUST 1986

FINAL REPORT FOR PERIOD MAY 1982 - MAY 1985

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SECURITY CLASSIFICATION OF THIS PAGE

AD-A172 749

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION Unclassified	1b. RESTRICTIVE MARKINGS None
2a. SECURITY CLASSIFICATION AUTHORITY None 2b. DECLASSIFICATION / DOWNGRADING SCHEDULE None	3. DISTRIBUTION / AVAILABILITY OF REPORT Approved for public release; distribution is unlimited.
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S) AFATL-TR-86-56
6a. NAME OF PERFORMING ORGANIZATION University of Kentucky 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION Air Force Armament Laboratory Munitions Division
6c. ADDRESS (City, State, and ZIP Code) Lexington, KY 40506	7b. ADDRESS (City, State, and ZIP Code) Eglin Air Force Base, FL 32542-5434
8a. NAME OF FUNDING / SPONSORING ORGANIZATION Air Force Armament Laboratory 8b. OFFICE SYMBOL (If applicable) MNW	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER F08635-82-K-0324
8c. ADDRESS (City, State, and ZIP Code) Eglin Air Force Base, FL 32542-5434	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. 61102F; PROJECT NO. 2307; TASK NO. E2; WORK UNIT ACCESSION NO. 69

11. TITLE (Include Security Classification)
On the Penetration of Semi-Infinite Targets by Long Rods (U)

12. PERSONAL AUTHOR(S)
Dr Stanley E. Jones, Dr Peter P. Gillis

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Final	FROM May 82 TO May 85	August 1986	29

16. SUPPLEMENTARY NOTATION

17. COSATI CODES

FIELD	GROUP	SUB-GROUP
19	04	
20	11	

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
Long-Rod Penetration
Terminal Ballistics

19. ABSTRACT (Continue on reverse if necessary and identify by block number)
The one-dimensional eroding-rod penetration theory proposed by Tate is modified in two ways. In the equation of motion of the rigid end of the rod, a proper accounting is made of mass transfer into the plastic region. Also, the mushroom strain at the deforming end of the rod is incorporated into the analysis. It is shown that this latter factor has a very substantial effect on calculated penetrations.

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION Unclassified
22a. NAME OF RESPONSIBLE INDIVIDUAL Lt Edward Schrank	22b. TELEPHONE (Include Area Code) 904-882-2141 22c. OFFICE SYMBOL AFATL/MNW

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE
UNCLASSIFIED

PREFACE

This report describes a new one-dimensional theory for the penetration of a semi-infinite solid by a long rod. Work was performed under sponsorship of the Bombs and Warheads Branch (now the Clusters and Warhead Branch) of AFATL through Contract No. F08635-82-K-0324.

The work reported here was performed during the period May 17, 1982 - May 16, 1985 under direction of the senior author, Prof. S. E. Jones of the University of Kentucky.

The authors wish to thank Dr. Joseph C. Foster, Jr. and Mr. Leo L. Wilson of the Clusters and Warheads Branch (MNW) for helpful guidance during the course of this work.

This report is expected to be published in the Journal of the Mechanics and Physics of Solids.

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LIST OF FIGURES

LIST OF SYMBOLS

A	initial cross-sectional area of rod
a,b	parameters defined by Equations (21) and (22)
c	$[Y/\rho]^{1/2}$
e	engineering strain
F	internal force
f,g,h	functions
L	initial length of rod
ℓ	remaining undeformed length of rod
P	interface force
p	interface pressure
R	target strength
t	elapsed time since impact
u	speed of penetration into target
V	initial impact speed of rod
v	current speed of rod
X	length of rod that has been consumed
Y	rod strength
z	penetration depth
Δ	increment
λ^2	ratio of target strength to that of rod
μ^2	ratio of mass density of target to that of rod
ξ	$\dot{\ell}^2$
$\bullet$	derivative with respect to time

SUBSCRIPTS

e	$z_e = z_f - z_o$

LIST OF SYMBOLS (concluded)

f	condition when penetration is finished
o	condition when $u = 0$

SECTION I

INTRODUCTION

For quite some time, there has been interest in the penetration of semi-infinite targets by long rods and jets. Numerous experimental and numerical studies of the problem have been carried out over the past four decades. The numerical studies have now evolved into gigantic hydrocodes which attempt to account for every detail of the event. These codes are expensive to run and do not offer much engineering insight into the penetration process. It is for this reason that the one-dimensional, eroding-rod model of Tate (References 1 and 2) is both a popular and useful tool for describing the event. For nearly two decades, Tate's theory has been regarded as the foundation for simple engineering modeling of the penetration process.

In this paper, a generalization of the basic equation of motion of the rod is made which leads to a modification of Tate's theory. The modified theory accounts for the expansion of the penetrator tip into a mushroom of diameter greater than that of the original rod, and provides an accurate treatment of mass transfer from the undeformed segment. Comparison with Tate's theory is made when appropriate.

SECTION II

THEORY

1. GENERAL

The approximate, one-dimensional analysis of the penetration of a half-space by a long (slender) rod is best explained for the case in which the penetration is accompanied by consumption of the rod. The other two cases, impact without penetration and rigid-body penetration, can be considered as special cases of the more general situation of the penetrator being consumed as it penetrates.

Figure 1 shows the penetrator, a cylindrical rod of initial length L, density ρ, and cross-sectional area A. Let V denote the initial velocity with which it impacts the semi-infinite target. After some time t has elapsed, the rod has penetrated the target to some depth z. In the process, a portion of the rod of length X has been consumed. The remaining portion of the rod has length L-X, denoted by ℓ; it is assumed to move as a rigid body with remaining velocity v. The speed of penetration into the target, $\dot{z}$, is denoted by u. Here the superposed dot indicates derivative with respect to time.

Figure 2(a) shows the rigid end of the rod at time t. An instant later, at time $t + \Delta t$, a portion of the rod end has been consumed by the penetration process and the remaining rod end has been decelerated. This is depicted in Figure 2(b). From these figures a simple, one-dimensional impulse-momentum equation can be written. Because the internal forces F are equal and opposite, only the external force P contributes to the impulse. As it is opposite to the directions of u and v, its contribution is negative: $-P\Delta t$. The total momentum at $t + \Delta t$ is $\rho A \Delta X u + \rho A(L-X-\Delta X)(v+\Delta v)$. At t the

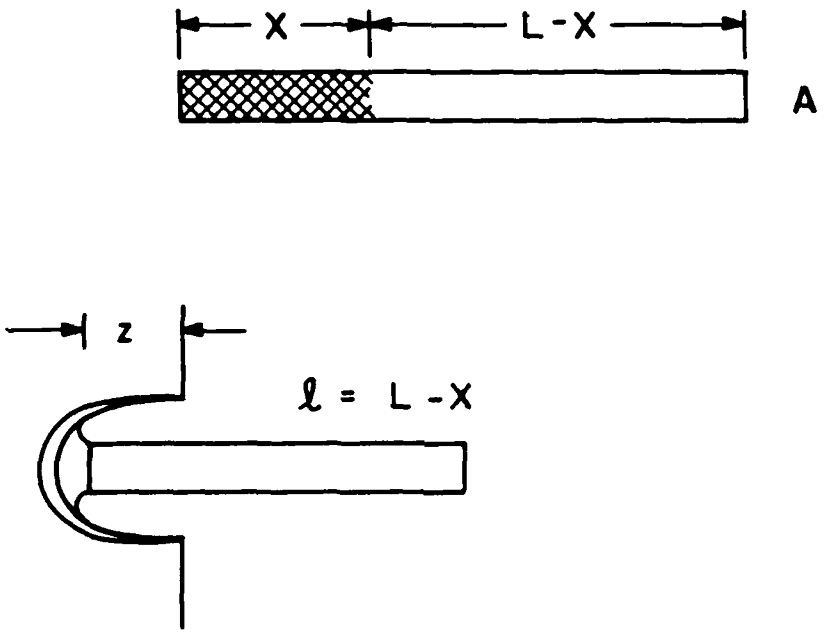

Figure 1. Schematic of Rod: (a) Shows Plastic Portion X, and Undeformed Portion L - X = ℓ; (b) Shows Penetration into Target to a Depth z.

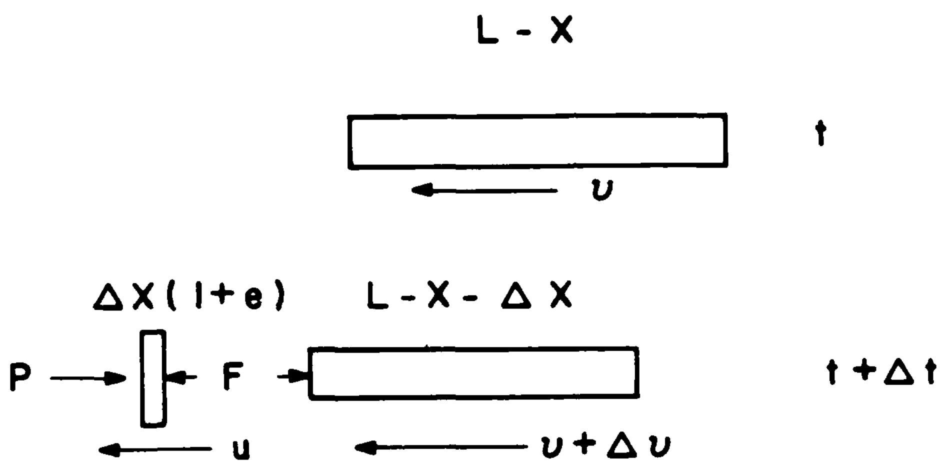

Figure 2. Schematic of the Transfer of Mass Element $\rho A \Delta X$ from the Undeformed to the Plastic Portion of Rod

momentum was $\rho A(L-X)v$. Subtracting to obtain the momentum change and equating that to the impulse gives

$$-P\Delta t = \rho A\{(L-X)\Delta v-\Delta X(v-u)-\Delta X\Delta v\} \tag{1}$$

Dividing through by Δt and taking limits as Δt approaches zero gives

$$-P = \rho A\{(L-X)\dot{v}-\dot{X}(v-u)\} \tag{1a}$$

as the equation of motion for the rigid end of the rod. From the definition $\ell = L-X$, Equation (1a) can be recast in the form

$$-P = \rho A\{\ell\dot{v}+\dot{\ell}(v-u)\} \tag{1b}$$

Again referring to Figure 2(b), it is assumed that a reasonable approximation to the external force is given by

$$P = pA/(1+e) \tag{2}$$

Here e denotes the engineering strain in the deformed element of the rod so that $A/(1+e)$, assuming isochoric deformation, is its current cross-sectional area; and p denotes the interface pressure at the penetrator tip. Substitution of Equation (2) into Equation (1b) gives

$$\ell\dot{v} + \dot{\ell}(v-u) = -p/\rho(1+e) \tag{3}$$

This is the rod equation of motion.

With reference to Figure 2, during the time interval Δt the back end of the remaining penetrator moves a distance $v\Delta t$ while its front end moves $u\Delta t$. The consequent change of length $\Delta\ell$ is $(u-v)\Delta t$ so that

$$\dot{\ell} = -(v-u) \tag{4}$$

This is a purely kinematical relationship among the variables.

To complete the statement of the problem requires the specification of functional relations for u, p, and e. Suppose, for example, that the penetration velocity u is some function of the rod speed v, say $u=f(v)$. It is frequently assumed that the pressure p is a function of penetration velocity; in this case p is also a function of v: $p = p(u) = p(f(v)) = g(v)$.

The strain e can be assumed to depend upon pressure (or stress) p; then it too is a function of v: $e = e(p) = e(g(v)) = h(v)$. The rod equation of motion can now be written as

$$\ell\dot{v} - [v-f(v)]^2 = -g(v)/\{\rho[1+h(v)]\} \qquad (5)$$

Equation (5) can be solved generally because the variables are separable. By making the substitution $\dot{v} = -(dv/d\ell)[v-f(v)]$ and rearranging, Equation (5) becomes

$$-\ell(dv/d\ell) = [v-f(v)]-g(v)/\{\rho[1+h(v)][v-f(v)]\} \qquad (6)$$

Separating variables and integrating from the starting conditions $\ell = L$ and $v = V$ leads to

$$\ell n(\ell/L) = \int_V^v \frac{dv}{g(v)\{\rho[1+h(v)][v-f(v)]\}^{-1} - [v-f(v)]} \qquad (7)$$

This is the general dependence of remaining rod length upon remaining rod speed.

Several special cases can be derived from the foregoing general theory. For example, the Tate analysis (References 1 and 2) is obtained by assuming that (v-u) and e are negligible in Equation (3), and that p is given by a modified Bernoulli equation

$$p = (1/2)\mu^2\rho u^2 + R = (1/2)\rho(v-u)^2 + Y \qquad (8)$$

Here $\mu^2\rho$ denotes target density, and R and Y are the respective strength factors of target and penetrator. In general these can be quite different from the static yield strengths of the two materials. Strain rate effects tend to increase these strengths while thermal softening tends to decrease them. Equation (8) is subject to the restrictions that: the right hand equality is invalid when the rod penetrates as a rigid body; both are invalid when there is no penetration. Unsteady flow is entirely neglected in Equation (8) and at present there is no satisfactory way of estimating the significance of this omission.

In this paper a more general form of the Tate analysis will be treated. It is assumed that Equation (8) gives the stagnation point pressure with sufficient accuracy, but (v-u) and e are retained in Equation (3). Inasmuch as Equation (8) arises from pseudo-steady-state considerations, it seems commensurate to assume that e is simply constant. This assumption is made for mathematical convenience and from ignorance. While it is expected that the appropriate value for e will grow larger in some manner with V, the details of any such relationship remain to be developed. However, treating e as some constant in any given impact is in keeping with the other steady-state assumptions.

Combining Equations (3), (4), and (8) gives the rod deceleration as either

$$\dot{v} = [(1+e)(v-u)^2 - (1/2)\mu^2u^2 - \lambda^2c^2]/\ell(1+e) \tag{9}$$

or

$$\dot{v} = [(1/2+e)(v-u)^2 - c^2]/\ell(1+e) \tag{9a}$$

Here λ^2 denotes the ratio R/Y and c^2 the quantity Y/ρ which has the dimensions of velocity squared. Equations (4), (8), and (9) or (9a) comprise a system of three equations in three unknowns: u, v, and ℓ. In the process of solving them, however, another equation

$$\dot{z} = u \tag{10}$$

should be utilized in order to find the most important element in the problem, viz., the penetration depth. The problem now is completely formulated. The parameters embedded in it are conveniently taken as μ^2 = (target density/penetrator density), λ^2 = (target strength/penetrator strength), $c^2 = Y/\rho$, e = engineering strain in the deformation zone of the penetrator, and V = the initial speed of impact of the rod. No general solution beyond Equation (7) is available for the foregoing set of equations; they must be integrated numerically for particular values of the parameters.

Before any calculations are attempted, the possibilities mentioned earlier, failure to penetrate and penetration without rod consumption, must be considered. Intuitively, the former would be associated with large values of μ^2, large values of λ^2 and/or small values of V; and the latter would correspond to μ^2 and/or λ^2 being small.

2. NO PENETRATION

Equation (8) can be used to find u in terms of v, μ^2, and c^2. Negative values of u imply no penetration. In this case set u = 0 and p = $(1/2)\rho v^2 + Y$. Then from Equation (9a)

$$\dot{v} = [(1+e)v^2-\lambda^2c^2]/\ell(1+e) \qquad (11)$$

From Equation (4) $v = -\dot{\ell}$ and $\dot{v} = -\ddot{\ell}$ which can be substituted into Equation (11) to obtain

$$\ell\ddot{\ell} + \dot{\ell}^2 = \lambda^2c^2/(1+e) \qquad (12)$$

The solution of Equation (12), taking account of the initial conditions $\ell = L$ and $\dot{\ell} = -V$ at t = 0, is

$$\ell_f/L = [1-V^2(1+e)/\lambda^2c^2]^{1/2} \qquad (13)$$

where ℓ_f denotes the length of rod remaining after it comes to rest. For initial velocities large enough that $V^2(1+e) > \lambda^2c^2$ Equation (13) will not apply. Instead the rod will be entirely consumed against the face of the target.

3. NO ROD DEFORMATION

Values of u larger than v imply rigid body penetration. In this case set $u = v$ and $p = (1/2)\,\mu^2\rho u^2 + R$. Then from Equation (9)

$$\dot{u} = -[\lambda^2 c^2 + (1/2)\mu^2 u^2]/\ell(1+e) \tag{14}$$

Now Equation (4) shows that $\dot{\ell} = 0$ so that ℓ remains constant. Noting that $\dot{u} = u(du/dz)$ Equation (14) can be readily integrated to obtain

$$z = z_o + [\ell_o(1+e)/\mu^2]\ell n\,[(2\lambda^2 c^2 + \mu^2 u_o^2)/(2\lambda^2 c^2 + \mu^2 u^2)] \tag{15}$$

Here, following Tate (Reference 2), u_o denotes the rod velocity at which $u = v$ first occurs and ℓ_o is the corresponding length. Also, z_o denotes the penetration to this point. The increment of penetration, z_e, that occurs in bringing the rigid rod to rest is obtained by substituting $u=0$ into Equation (15). Thus, the total penetration depth is given by

$$z_f = z_o + [\ell_o(1+e)/\mu^2]\ell n[1 + \mu^2 u_o^2/2\lambda^2 c^2] \tag{16}$$

Note that this result is almost identical to Tate's and has the general form of the Petry equation (Reference 3) or the DeMarre formula (Reference 4). For the situation in which rigid body penetration proceeds from initial impact, $u_o = V$, $\ell_o = L$, and Equation (16) yields

$$z_f = [L(1+e)/\mu^2]\ell n[1+\mu^2 V^2/2\lambda^2 c^2] \tag{17}$$

4. PENETRATION ACCOMPANIED BY ROD DEFORMATION

Having disposed of the rigid-body and zero penetration cases, the penetration-with-consumption case is resumed. The value of u obtained from Equation (8) depends upon the parameters μ^2 and λ^2. When either of these is unity, the equation is essentially linear; otherwise it is quadratic and rather more complex. Consider first the condition of equal strengths of target and rod; here $R = Y$ so that $\lambda^2 = 1$ and Equation (8) yields

$$u = v/(1+\mu) \tag{18}$$

In this situation u is always smaller than v and the two reach zero simultaneously. There is no rigid-body penetration phase under these conditions. From Equation (18) $(v-u) = \mu v/(1+\mu)$ so that Equation (4) can be written as

$$\dot{\ell} = -\mu v/(1+\mu) \tag{19}$$

Differentiating Equation (19) and substituting into Equation (9a) gives

$$\ell\ddot{\ell}+a\dot{\ell}^2 - b = 0 \tag{20}$$

where

$$a = \mu(1/2+e)/(1+\mu)(1+e) \tag{21}$$

and

$$b = \mu c^2/(1+\mu)(1+e) \tag{22}$$

By defining $\xi = \dot{\ell}^2$, from which $d\xi/d\ell = 2\ddot{\ell}$, Equation (20) can be rewritten as

$$\ell(d\xi/d\ell) + 2a\xi - 2b = 0 \tag{23}$$

This equation is linear and can be simply integrated subject to $\xi(L) = \mu^2V^2/(1+\mu)^2$.

Two cases develop: first, $a=0$, which implies $e=-1/2$. In this case Equation (23) integrates to

$$\xi = \dot{\ell}^2 = (\mu^2V^2)/(1+\mu)^2 - 2b\ln(L/\ell) \tag{24}$$

From Equation (24) it can be seen that there will always exist a non-zero, unconsumed rod length ℓ_f corresponding to $\dot{\ell} = 0$ and given by

$$\ell_f = L\exp\{-\mu^2V^2/2b(1+\mu)^2\} = L\exp\{-\mu V^2/c^2(1+\mu)\} \tag{25}$$

For this case in which u is directly proportional to v, Equation (18), $\dot{\ell}$ is also proportional to v, Equation (19). From Equations (18) and (19) $u = -\dot{\ell}/\mu$ so that the penetration depth, given by Equation (10), in this case becomes

$$z = \int_0^t u\,dt = -1/\mu\int_0^t \dot{\ell}\,dt = (L-\ell)/\mu \tag{26}$$

The total depth of penetration, z_f, is given by

$$z_f = (L-\ell_f)/\mu \tag{26a}$$

where ℓ_f is given in Equation (25).

The more important case is $a \neq 0$. For this case, ℓ^{2a-1} is an integrating factor of Equation (20) and the solution is

$$\xi = \dot{\ell}^2 = b/a + [\mu^2V^2/(1+\mu)^2 - b/a](L/\ell)^{2a} \tag{27}$$

If a is negative, then the penetration process will always be complete before the rod is completely consumed by plastic deformation. The final undeformed length ℓ_f is obtained from the solution of

$$(\ell_f/L)^{2a} = 1 - a\mu^2V^2/b(1+\mu)^2 \tag{28}$$

The total depth of penetration is given by Equation (26a) with ℓ_f given in Equation (28).

For positive values of a, the situation is somewhat more complicated. If $b/a = 2c^2/(1+2e) > \mu^2V^2/(1+\mu)^2$, then penetration ceases before the rod is completely consumed by plastic deformation. The final undeformed length is still given by Equation (28) with the total depth of penetration again given by Equation (26a).

The only remaining possibility is $a>0$ with $b/a = 2c^2/(1+2e) <$ $\mu^2V^2/(1+\mu)^2$. In this case, the rod will be completely consumed by plastic deformation before penetration ceases. The total penetration depth is then

$$z_f = L/\mu \tag{29}$$

according to Equation (26a).

The other case in which Equation (8) is linear in u is for $\mu^2 = 1$. In this case

$$u = (1/2)\ v + (1-\lambda^2)c^2/v \tag{30}$$

For u to be positive requires a rod velocity such that

$$(1/2)v^2 + (1-\lambda^2)c^2 > 0 \tag{31}$$

Condition Equation (31) will always be satisfied for $\lambda^2 < 1$ until v decreases to zero. For $\lambda^2 > 1$ there is a critical initial striking speed $[2(\lambda^2-1)]^{1/2}$ c. For actual initial impact velocities less than this value, penetration will not occur. For impact speeds greater than this critical value, penetration-with-consumption will occur and will continue until the rod is decelerated to $[2(\lambda^2-1)]^{1/2}$ c when penetration will cease.

For u to be smaller than v in this case ($\mu^2 = 1$) requires that

$$(1/2)v^2 - (1-\lambda^2)c^2 > 0 \tag{32}$$

Here the inequality will always be satisifed for $\lambda^2 > 1$ but, from the discussion above, this will include situations of zero penetration. For $\lambda^2 < 1$ there is a critical initial striking speed $[2(1-\lambda^2)]^{1/2}$ c below which rigid body penetration occurs and Equation (17) applies. For initial striking speeds above this critical value there will be some penetration-with-consumption until the rod is decelerated to $[2(1-\lambda^2)]^{1/2}$ c after which some further rigid body penetration occurs, for which Equation (16) applies.

When $\mu^2 \neq 1$ and $\lambda^2 \neq 1$ Equation (8) is quadratic in u and has the solution

$$u = \{v - [\mu^2v^2 - 2(1-\mu^2)(1-\lambda^2)c^2]^{1/2}\}/(1-\mu^2) \tag{33}$$

Whenever $v > [2(1-\mu^2)(1-\lambda^2)]^{1/2}c/\mu$, the root in Equation (33) is real and the rod penetrates the target. The complexity of this relationship between u and v precludes the possibility of finding any exact solutions to the differential equations beyond that given in Equation (7). The system is thus integrated numerically.

5. LIMIT AT HIGH IMPACT SPEEDS

For v^2, sufficiently large in comparison to c^2, Equation (33) gives the approximate result

$$u = v/(1+\mu) \tag{34}$$

which is identical to Equation (18). In this case Equation (19) applies, and Equations (26) and (26a) follow. Rearranging Equation (26a) gives

$$\mu z_f/L = 1 - \ell_f/L \tag{26b}$$

At the highest impact velocities the penetrator is completely consumed ($\ell_f = 0$) and from Equation (26b) it follows that the limit of $\mu z_f/L$ is unity as V/c becomes very large. This is obvious from the numerical calculations shown in the figures.

SECTION III

RESULTS AND DISCUSSION

Figure 3 shows some calculated results for cases favorable to penetration. In these cases, by comparison to the target the penetrator is stronger ($\lambda^2 = 1/3$) and more dense ($\mu^2 = 0.64$). This figure shows the effect of one element introduced by the present theory, viz., the mushroom strain at the projectile end. As can be seen from Equations (9) and (9a) this strain, which is negative, directly increases the projectile deceleration. The obvious physical interpretation is that the incoming projectile momentum is resisted by a larger target area. For zero strain the results of the present analysis are nearly identical to those calculated from Tate's analysis. However, as the impact end strain is assigned increasingly larger magnitudes, the penetration depth is greatly reduced. It must be concluded that this mushroom strain plays a highly significant role in the penetration process.

Figure 3 shows that for strains smaller in magnitude than 0.5 the penetration at first increases with impact velocity, reaches a maximum, and then decreases. This is observed in the calculations based upon Tate's theory also, and has been discussed at length in his 1969 paper (Reference 2). Based upon the present analysis, it seems likely that when the proper dependence of e upon V is discovered, penetration depth will be predicted to increase monotonically with impact speed.

Figure 4 shows some calculated results for cases in which the penetrator and target are of identical materials. They have equal strengths ($\lambda^2 = 1$) and densities ($\mu^2 = 1$). This figure shows the effect of the second major element introduced by the present theory, viz., the relative velocity term in the equation of motion. This is most easily seen in Equation (9a).

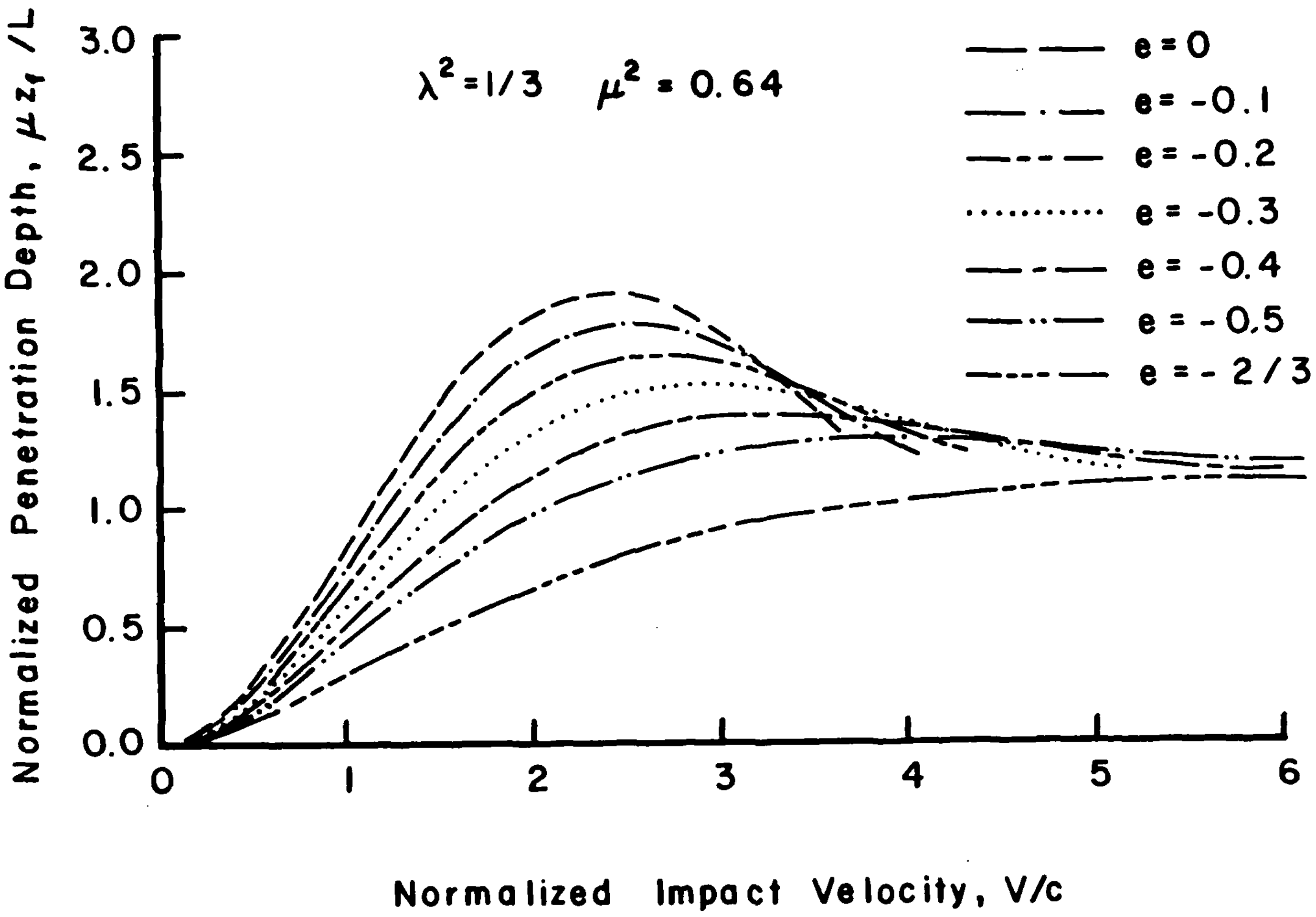

Figure 3. Normalized Penetration Depth vs. Normalized Impact Velocity for Cases in which $\lambda^2 = 1/3$ and $\mu^2 = 0.64$

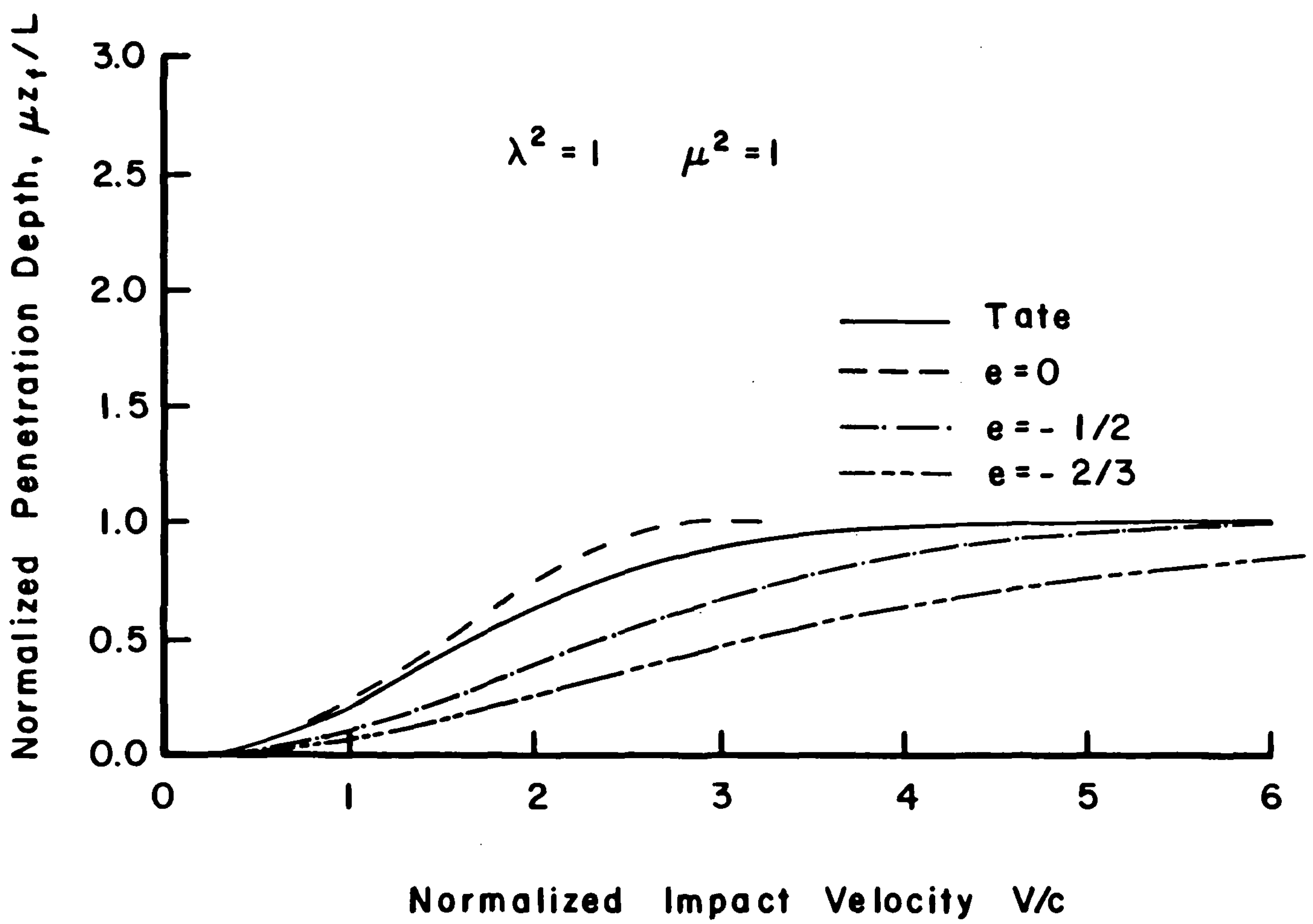

Figure 4. Normalized Penetration Depth vs. Normalized Impact Velocity for Cases in which $\lambda^2 = 1$ and $\mu^2 = 1$

Here the term $-c^2$ that occurs in Tate's equation is reduced by the term $(1/2 + e)(v - u)^2$ in the present theory. For $e = 0$, Figure 4 shows that substantially larger penetrations are calculated from the present theory than from the Tate theory over the range of impact velocities bridging between rigid body penetration and the limiting case at high velocities. This effect is not so strong as the mushroom strain effect, however, and the figure shows a reverse to smaller penetration depths at two non-zero strains.

Figure 5 shows some calculated results for cases unfavorable to penetration. In these cases, by comparison to the target the penetrator is weaker ($\lambda^2 = 3$) and less dense ($\mu^2 = 1.44$). This figure shows that for $e = 0$, calculations based upon the present theory are virtually identical to those based on Tate's theory. But there remains a highly significant effect due to impact face strain.

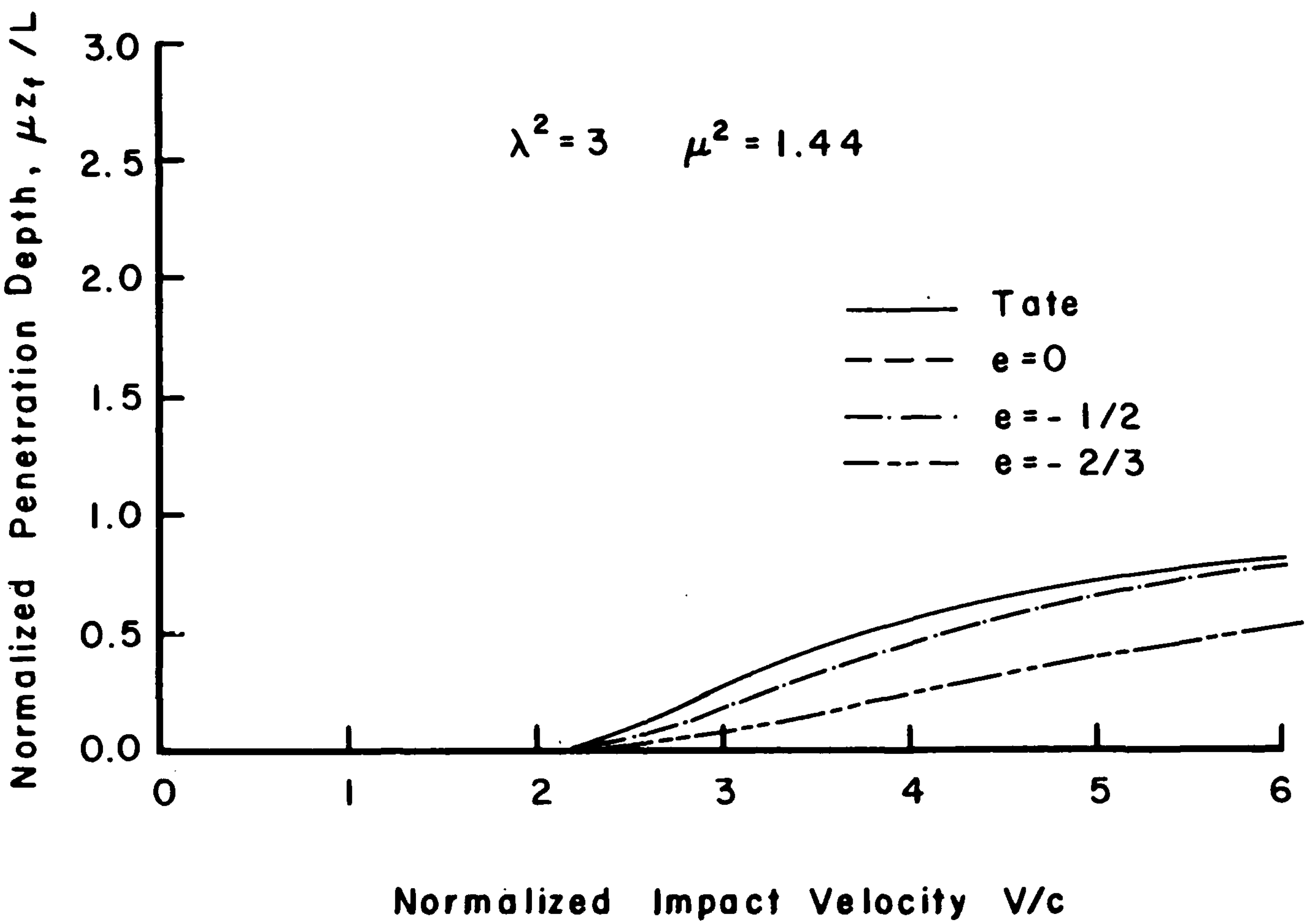

Figure 5. Normalized Penetration Depth vs Normalized Impact Velocity for Cases in which $\lambda^2 = 3$ and $\mu^2 = 1.44$

SECTION IV

CONCLUSIONS

The present analysis provides a one-dimensional penetration theory that is more accurate than its antecedent.

Two new effects are introduced in the present theory: the change of momentum of material crossing the plastic interface, and the strain discontinuity at this interface. Of these two, the latter has the greater impact on calculated results.

The present analysis is capable of describing impact penetration data without recourse to abnormally large strength parameters for the target and penetrator materials.

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